From Lucas Sequences to Lucas Groups

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I study Lucas Sequences, recursive sequences that generalize the Fibonacci Sequence, and Lucas Groups, solutions of a variant of Pell's Equation mod p. Lucas Sequences have a broad background in the literature, but Lucas Groups have seldom been studied before. When taken mod some prime p, Lucas Sequences form a cyclic group, which will be a subgroup of the corresponding Lucas Group. I show exactly when these two groups are isomorphic by relating a constant associated with the Lucas Sequence with primitive elements of the field of order p. Furthermore, I prove how often this isomorphism occurs for each prime p > 3. I also give a computationally efficient method to find the index of the Lucas Sequence in the Lucas Group when they are not isomorphic. Then, I present several novel primality tests based on Lucas Groups. I generate data, comparing the accuracy of these primality tests to current tests based on Lucas Sequences, and find that the accuracy of our new tests far surpasses the accuracy of current primality tests.

Awards Won:
Fourth Award of $500
American Mathematical Society: Certificate of Honorable Mention