Extreme Point in the Triangle Plane

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The present work is used to set a mathematical problem to discover absolute barycentric coordinates of locus P^n_{min} in the triangle plane, featured with the n^{th} powers of distances from points of this locus to the straight lines containing the triangle sides having minimum sum value. The accomplished research was used to resolve this problem, in the course of which, the search of barycentric coordinates of the extremum point, for $n\geqslant 1$ has been discovered. It's shown, that in case $n=1$ the answer for the problem depends on type of the triangle. In the case $n>1$ P^n_{min} is the single point, which lies strictly inside the triangle and having absolute barycentric coordinates: $$ \left(\begin{array}{lcr} \displaystyle \frac{a^\frac{n}{n-1}}{a^\frac{n}{n-1} + b^\frac{n}{n-1} + c^\frac{n}{n-1}}; & \displaystyle \frac{b^\frac{n}{n-1}}{a^\frac{n}{n-1} + b^\frac{n}{n-1} + c^\frac{n}{n-1}}; & \displaystyle \frac{c^\frac{n}{n-1}}{a^\frac{n}{n-1} + b^\frac{n}{n-1} + c^\frac{n}{n-1}} \end{array} \right). $$ The theorem have cases which are gained for the first time: \begin{itemize} \item[1)] in regular triangle P^n_{min} coincides with its center for $n>1$; \item[2)] P^n_{min} for $n\rightarrow\infty$ coincides with incenter. \end{itemize} As the result of the research, for the first time, it was possible to gain expressions for the absolute barycentric coordinates of the extremum point. The research may be used in the optimization theory and physics.