The Frequency and Distribution of Consecutive Quadratic Residues Modulo p

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First, the number of pairs of consecutive quadratic residues modulo an odd prime p are counted, using sums of the Legendre symbol. We then extend this to triples of consecutive quadratic residues modulo an odd prime p. We find explicit formulas for the number of such triples based on the value of p (mod 8), using Jacobsthal's theorem. We then show the existence of infinitely long sequences of consecutive quadratic residues for all sufficiently large primes p. We examine the sum of the Legendre symbol of n(n+1)(n+2)(n+3) as it ranges across all of Z/pZ using the Riemann-Hurwitz theorem and the Hasse-Weil bound, and make some conjectures as well. We extend this to even-degree polynomials and squarefree quartics.