Asymptotics of Character Sums

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In this project, we aim to prove certain properties about a particular function c(n) = b_nr(n). This is where b_n is a Boolean function with b_n being 1 if n = x^2 + y^2 for some integers x and y or 0 otherwise and r_chi(n) is the sum of all of the Dirichlet characters which are divisible by n. Since chi will be clear from context, we will suppress the subscript throughout this manuscript. The function c(n) sums the all of the chi values of the divisors of a certain number n if and only if n can be expressed as the sum of two squares. Therefore, the question we ask is the following: What are the asymptotics of the character sums of the function c(n)? In order to investigate this problem, we first represent the character sum of r(n) as an asymptotic and prove that the asymptotic is roughly L(1, chi) with a small error term. Additionally, we compute a representation for the character sum c(n) as an Euler product, and also find error bounds on the asymptotic for the character sum.

Awards Won:
Fourth Award of $500
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