Mathematics of Gene Regulation: Control Theory for Ternary Monomial Dynamical Systems

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A finite dynamical system (FDS) is a function f: F_m^n?F_m^n, where F_m^n is a set of m^n elements. Due to its applications to gene regulatory networks, it is desirable to develop mathematical methods to obtain information about the regulatory iterations between genes, particularly when these iterations eventually stabilize. The researchers presented a formalization of the idea of a “stabilizable” discrete dynamical control system. Furthermore, the necessary and sufficient conditions for a Boolean monomial dynamical control system to be stabilizable in terms of properties of the dependency graph associated with the system were considered. Utilizing prior results, the focus of this research was to extend the understanding of monomial control systems by studying ternary systems, that is, the case m=3. In this work, a language concerning the dependency graph of ternary systems was formalized; additionally, the researchers presented and demonstrated sufficient conditions for the state space of a ternary monomial dynamical system to be a fixed-point system regarding the dependency graph. The importance of the latter point arises from the use of graphs instead of linear matrices to determine stability. Moreover, the researchers showed sufficient conditions for a ternary monomial control system to be stabilizable and an algorithm to stabilize these systems. The results extend the applicability of a control theory for FDS with m=3 to address the complex interactions between components in real-life systems- such as gene regulation where the genes have three distinct levels of expression.

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