On the Modular Properties of Hypothetical Collatz Loops

Park, Jian (School: Sage Hill School)

The Collatz function is defined as follows: start with a positive integer x. If x is odd, multiply it by three and add one, and if x is even, divide it by two. A recursion of this function creates a Collatz sequence, and the Collatz conjecture predicts that no matter what initial value x is chosen, any Collatz sequence will eventually reach 1. One possible scenario that would disprove this conjecture is if there existed a loop that did not include the number 1. In order to explore these hypothetical loops, I first developed the Collatz modulo web, which is a model that can compute the possible modulo values of the elements in a general Collatz sequence. In order for a loop to be real, it must also exist within the modulo webs, so I developed a depth-first search algorithm that traversed all possible trajectories within the modulo webs to find potential loops with length n. After, these potential loop numbers were applied to a general Collatz formula to check for existence. The Collatz modulo web concept can predict the modulo values of Collatz sequence numbers, and combined with the searching algorithm, it can be used to computationally calculate the existence of loops by loop length, rather than by just initial value. This new method to check for loops may provide new insight into the previously unsolved problem.

Awards Won: