Finnish Baseball's Draw of Choice (Hutunkeitto) as a Two Player Finite Game

Jarviniemi, Olli (School: Valkeakosken Tietotien Lukio)

 $textit{Draw of choice} (Finnish: "hutunkeitto") in Finnish baseball can be seen as a finite two-player game with winning and losing positions. By inspecting winning and losing positions one can determine the winning moves if such exist. Let the players be A and B. \textit{Draw of choice} can be stated as follows: let positive reals x, a_1, a_2, b_1, b_2 be given. A begins and subtracts some number from the interval $[a_1, a_2]$ from x. After this B subtracts some number from the interval $[b_1, b_2]$ from the current number. This procedure is continued until the number is negative or 0. The player to make the last move wins. Following results are found: let $k \geq 0$ be an integer. If x satisfies $k(a_1 + b_2) < x \le (k+1)a_2 + kb_1$ A wins. A loses if $(k+1)a_2 + kb_1 < x \le (k+1)(a_1 + b_2)$. Similar results hold for B. Also, if $a_2 - a_1 > b_2 - b_1$, then A wins for all sufficiently large x. The solution is computationally efficient: determining the winner for a single value x can be done by finding the integer for which $k(a_1 + b_2) < x \le (k+1)(a_1 + b_2)$ holds. After this it should be checked if $x \le (k+1)a_2 + kb_1$ holds. If yes, A wins, and otherwise A loses.$