Looking at the Dynamics of Q(Z)=Z^2+C in the Space of 2X2 Real Matrices

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The dynamics of polynomials in the space of 2X2 real matrices of the form Q(Z) = Z^2+C with a Constant C consisting of complex eigenvalues, identical eigenvalues, and distinct real eigenvalues are observed. The dynamics of the 3 types of maps are compared with each other, with a focus on cycles that are not confined by the invariant plane. Additionally, the dynamics of these matrix maps are compared with maps of the form P(z)=z^2+c where z and c are complex numbers. The dynamics are experimentally compared by computing Julia sets for different values of C. Observing the sets created by C with complex eigenvalues and C having identical real eigenvalues yields identical results. Whenever a cycle off the invariant plane occurs in the complex case, a cycle off the invariant plane occurs in the identical real case. Contrarily, dynamics in spaces of distinct real eigenvalues do not match up with the case of complex eigenvalues. Cycles off the invariant plane do not occur in the real eigenvalue case whereas they are common in the complex eigenvalue case. We are also able to find similarities between the complex Mandelbrot set and the dynamics of these matrix functions.

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