Braid Groups on Triangulated Surfaces and Singular Homology

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The mathematical field of algebraic topology studies the association of algebraic data to topological spaces, which are generalizations of geometric shapes. Since its creation in the early 20th century, algebraic topology has shown to be a powerful tool in both pure mathematics (in particular in geometry and topology) and in data science (e.g detecting patterns in large data sets). My research finds a new connection between two different algebraic structures that can be associated to a surface, namely its homology groups (counts holes in the surface) and its surface braid groups (describes the movement of particles on the surface). In particular, I constructed a natural surjective group homomorphism from surface braid groups on any closed orientable surface to its first singular homology group with integral coefficients. I proved that if the surface is triangulated (approximated by gluing together triangles), then the kernel of this map is generated by elementary braids which arise from the triangulation of the surface. My results indicate that triangulations of surfaces determine useful information about the structure of their surface braid groups, and in particular that braid groups on triangulated surfaces completely determine the first singular homology groups of such surfaces. Furthermore, my results have applications in algebraic/geometric topology, group theory, braid theory, and theoretical physics.

Awards Won:

Second Award of $2,000

American Mathematical Society: Certificate of Honorable Mention