

# Mersenne Primes: An Exploratory Study of Patterns and Some New Conjectures

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\*Introduction Mersenne Primes:- A number of the form  $2^n - 1$  where  $n$  is a natural number is a Mersenne number. If its value is prime then it is known as a Mersenne Prime. Mersenne primes are named after the mathematician Marine Mersenne who discovered these around 1550 BC with the help of Perfect Numbers\*. Some of the initial Mersenne primes were –  $2^2-1=3$  ;  $2^3-1=7$  ;  $2^5-1=31$  ;  $2^7-1=127$  ;  $2^{13}-1=8191$  and  $2^{17}-1=131071$  The Largest known Mersenne Prime Number is  $2^{(82,589,933)}-1$  and it contains 24,862,048 digits. It took approximately 12 months of calculations to find this Prime Number.

\*Methodology 1. In  $2^n - 1 = X$ , I have observed some patterns between the power of 2, that is  $n$ , and the number of digits in its value, that is,  $X$ . The patterns were: -  $2^{311} \sim 32 + 2 \times 31 = 94$  {i.e.  $2^{501}$  has 94 number of digits}  $2^{321} \sim 33 + 2 \times 32 = 97$  {i.e.  $2^{511}$  has 97 number of digits}  $2^{331} \sim 34 + 2 \times 33 = 100$  {i.e.  $2^{521}$  has 100 number of digits} 2. I have studied about the Euclid's proof of prime numbers which leads me to propose an easy method to find prime numbers. 3. I have formed a quadratic equation whose roots are the two prime numbers used in the RSA encryption system. \*Results and Findings 1. I have formed patterns and two relations between the values of  $2^n$ ,  $2^n - 1$  and their number of digits. They help me to eliminate 33.33% numbers which will never be a prime number. The relations are: - For Power:  $An = 1 + (n-1)10$  For number of digits: -  $An = 1 + (n-1)3$  2. I have proposed a new and easy method of predicting big prime numbers. 3. I am able to find the values for the two primes in the RSA encryption system non computationally. I am working on its computational method.

**Awards Won:**