On Stallings Geodesic Braids Conjecture

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In low-dimensional topology, one of the most beautiful and complex topics is braid theory. There are deep connections between braids, knots, mappings on surfaces, and even algebraic geometry. The key to these connections is the normal forms of braids. Braids on several strands form a group with standard Artin generators so that any braid has word representatives in these generators, corresponding to its plane diagrams. Minimal crossing number diagrams of braids correspond to geodesic words (in the sense of geometric group theory). There is a problem due to J. Stallings related to some kind of regularity in the language of geodesic words in braid groups. The main difficulty is that this language is complex and unclear. We present an elegant and natural description of geodesic words in braid groups based on the notion of homogeneous words. Homogeneous words were introduced by Stallings and also include positive, negative, and alternating words. We prove that homogeneous words are geodesic and rigid and that any geodesic word is built from them. It is remarkable that there is no description of minimal diagrams of knots or links. Using our results, we prove Stallings conjecture for all homogeneous words and other important classes of words. We also find statements that are equivalent to Stallings conjecture and we show that correctness of some fast algorithm, which was introduced recently without proof for braids on four strands, is equivalent to Stallings conjecture on four strands.

Awards Won:
American Mathematical Society: Third Award of $500