Finding Chebyshev-Type Functions

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This project is motivated by studying Chebyshev polynomials such as cos?(mx) is a polynomial of cos?(x). It is interesting to know if there are other non-constant continuous functions f having the similar property, i.e., f(mx) is a polynomial of f(x). In this study, we are able to characterize all non-constant continuous functions f from the positive real number set to the complex number set such that for all positive integer ??, there exist polynomial P(x) satisfying f(mx)=P(f(x)). Furthermore, we generalize the problem by replacing the positive integer set with an arbitrary subset of the positive integer set.

Awards Won:

Third Award of $1,000

American Mathematical Society: Second Award of $1,000