

Finding Chebyshev-Type Functions

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This project is motivated by studying Chebyshev polynomials such as $\cos^n(mx)$ is a polynomial of $\cos^n(x)$. It is interesting to know if there are other non-constant continuous functions f having the similar property, i.e., $f(mx)$ is a polynomial of $f(x)$. In this study, we are able to characterize all non-constant continuous functions f from the positive real number set to the complex number set such that for all positive integer n , there exist polynomial $P(x)$ satisfying $f(mx) = P(f(x))$. Furthermore, we generalize the problem by replacing the positive integer set with an arbitrary subset of the positive integer set.

Awards Won:

Third Award of \$1,000

American Mathematical Society: Second Award of \$1,000