

# On the Largest Axes-Parallel Rectangle among Points in a Square

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Given  $S$ , a set of  $n$  points contained in the unit square  $Q = [0, 1]^2$ , let  $f(S)$  denote the area of the largest axes-parallel rectangle that does not contain any of the points of  $S$  in its interior. Further, let  $f(n)$  be the minimum value of  $f(S)$  over all sets  $S$  of  $n$  points in  $Q$ . In 2009, Dumitrescu and Jiang proved that  $f(2) = (3 - \sqrt{5})/2$ ,  $f(4) = 1/4$ , and the following general bounds for  $f(n)$ :  $(1.25 - o(1)) \cdot 1/n \leq f(n) \leq 4 \cdot 1/n$ . We show that  $f(3) = 0.3079 \dots$ ,  $0.2192 < f(5) < 0.2215$ , and we improve the bounds in the general case:  $(1.31 - o(1)) \cdot 1/n \leq f(n) \leq 1.91 \cdot 1/n$ .

## Awards Won:

Third Award of \$1,000

American Mathematical Society: Third Award of \$500