The Bilman-Trogdon Inverse Scattering Transform for the Toda Lattice

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The Toda lattice is a Hamiltonian system which attempts to model a one-dimensional solid. The system was originally introduced by Japanese physicist Morikazu Toda in 1967, and is still relevant today due to the nature of its soliton solutions. Solitons are special waves that propagate at a constant velocity while maintaining their shape due to a balance of Kerr nonlinearity and dispersion effects. This special nonlinear nature makes solitons useful in fiber optics as they can transmit data without so many issues stemming from fiber loss (erbium specifically serves as a natural amplifier for soliton pulses.) This project focused on solving the Toda lattice using the inverse scattering transform method in order to identify and verify explicit soliton solutions. This project sought autonomous lattice solution generation and verification, and by extension looked to verify or refute the processes defined in Bilman and Trogdon's 2017 research paper, "Numerical Inverse Scattering for the Toda Lattice." The method relies upon the computation of scattering data and the deformation of a vector Riemann-Hilbert problem, which were found using a Mathematica Riemann-Hilbert package along with the Bilman-Trogdon inverse scattering transform package. The soliton solutions identified were then compared to the standard soliton model (sech^2(x) for single soliton pulses) using a MATLAB script to perform numerous statistical tests. The tests affirmed that the solutions found were valid solitons and proper solutions to the lattice. A secondary goal of the project was to provide understandable models and research in order to make the topic more accessible.

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