Dynamics of the Tangent Map

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I developed a new method to analyze expanding maps with indifferent fixed points. I applied this method to specifically analyze the iterations of the tangent map, proving both measure-theoretic and topological genericity of points with dense trajectories, and fully analyzing possible behaviors of trajectories for all starting points. Also, using the iterations of tangent and splitting the real numbers into intervals, I was able to construct a bijection between real numbers and sequences of whole numbers. Finally, using this bijection, I was able to prove a characteristic theorem regarding the sequences of iterations themselves, proving that the sets of points whose iterations fell into only specific intervals were Cantor sets. My numerical results show that the iterations are not uniformly distributed in the real numbers but rather follow a universal inverse square law, due to the indifferent fixed point x=0. The properties of arctan^(n)(x) allowed for a heuristic explanation of these results.

Awards Won:
Fourth Award of $500
American Mathematical Society: Second Award of $1,000