Contradictions in the Banach-Tarski Paradox within Euclidean Space

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This project focuses on a contradictory case within the Banach-Tarski paradox. The paradox outlines how any three-dimensional figure can be transformed into any other figure while only using translation and rotation. On the surface, transforming one figure into any other is paradoxical in nature and beyond most human experience but the paradox has never been disproven under its own constraints. Therefore, the research question that this project focuses on is as follows: “How can the Banach-Tarski paradox be disproved under the constraints outlined in the proof?” The proposed answer to this question is: “An analysis of specific examples and constructions within Euclidean space reveals that the Banach Tarski paradox is flawed.” The approach used in this investigation relies on the nature of fractal structure within Euclidean space. By analyzing the nature of volume and surface area with regards to a three-dimensional rendering of the famous Koch Snowflake, a flaw in the Banach – Tarski paradox was recognized. By disregarding cases of uncountably infinite surface area, the paradox fails to account for all points within certain structures. This claim was found to be properly supported within all constraints outlined in the Banach – Tarski paradox. Using geometry and sequences, a method for calculating the exact surface area and volume of a three-dimensional Koch Snowflake was found. This was then used in tandem with an appeal to Cantor’s diagonal argument to argue that the Banach – Tarski paradox fails to account for all points within a set to construct two copies of the three – dimensional Koch Snowflake.

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