

On a Generalization of Artin's Conjecture for Primitive Roots in Gaussian Integers

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We propose a generalization of Artin's conjecture on primitive roots to the ring $\mathbb{Z}[i]$ of Gaussian integers. We conjecture that for a fixed positive integer q , every non-zero Gaussian integer a that is not $+1$ or -1 , generates a cyclic subgroup of the multiplicative group of $\mathbb{Z}[i]/p$ of residue index q for infinitely many prime ideals p . In several special cases we reduce it either to the classical Artin's conjecture, or to its extension for near-primitive roots, the Golomb's conjecture. We divide the conjecture into three cases: when a is on the real axis, when a is on the imaginary axis, and when a is on neither axes. We conclude by showing that for every a , we have that the sum of $d(a,q)$ over all positive q equals 1, where $d(a,q)$ is density of the prime ideals p yielding subgroups of index precisely q .

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