Optiming Length of Planar Curves

Kloud, Vojtech (School: Prvni Soukrome JazykovE Gymnazium Hradec Kralove)

Optimization plays an essential role not only in mathematics, but in many related fields such as biology, robotics or transport. This particular research focuses on finding an optimal planar path between two endpoints that avoids given obstacles. The main goal is to show that a given curve, in a plane with obstacles, is the shortest one among all admissible curves. Basic concepts regarding this problem, namely the idea of an obstacle and an admissible curve are defined. Methods of calculus of variations and theorems of real analysis were employed. I have developed sensible definition of obstacles and the spaces of all admissible curves. Due to convexity of a length functional, I have characterized the problem and determined the existence of a solution in one of the considered cases. Further, I have considered some configurations of the given obstacles and determined a general form of shortest curves in these cases. The developed method allows to demonstrate that the resulting curve is indeed the shortest one and was utilized in particular cases. Furthermore, I have shown that the space of all admissible curves is a convex space. Hence, I may treat the problem as a problem of convex optimization. Along with this, I have developed a sensible notation which could be used in further research from an analytical perspective.

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