The Geometry of Free Nilpotent Groups

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A wonderful fact called Thurston's geometrization conjecture states that there are essentially only eight three-dimensional geometries. For the last 20 years, there are lots of attempts to describe high-dimensional geometries the way Thurston did. All works dedicated to solving this problem have been based on the idea of describing soluble and nilpotent groups Lie with corresponding manifolds. So one of the important steps in solving the problem is to understand the geometries of nilpotent groups. To describe geometries of nilpotent it’s sufficient enough to describe the geometry of free nilpotent groups. Large scale properties of these geometries are closely related to geometric properties of its lattice, so naturally, there is a problem of describing free nilpotent geometries that have been investigating for 30 years by mathematician. Most of their works are dedicated to understanding the simplest non-abelian case of free nilpotent groups - discrete Heisenberg group. In our research, we present a complete solution, fully describing the geometry of free nilpotent groups. More precisely, we give the solution in terms of a model that describes words as polygonal chains in corresponding metric spaces and giving criteria for two words representing the same element. Also, we showed the efficiency of our solution and created methods giving formula of length for elements in any free nilpotent group of finite rank and class 2 and describing some continuous results on free nilpotent Lie groups.

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