Analyzing Collatz-type Fractals Using Minkowski Dimensions

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The Collatz Conjecture can be written as a single-part function using indicator functions, and extending it to the complex plane allows for it to be graphed as a fractal. The indicator functions' purpose is to decide what operation is performed on an integer based on its congruence modulo k. In this research project, the researcher compared the original Collatz Conjecture to another Collatz-type problem to propose conjectures similar to the two. The researcher would write these problems as single-part functions, to then extend them to the complex plane, allowing them to be graphed as fractals. The researcher hypothesized that the fractals' minimum and maximum values for Minkowski dimensions would be within 0.1 of the original problem's. The researcher proposed problems involving congruence mod 4 and mod 8 and wrote holomorphic functions for these and the mod 3 problem using different types of indicator functions. The researcher, after generating and analyzing the fractals, found that the data outputted by the program for the mod 2 and mod 4 fractals was the same, possibly meaning that the fractals are indeed identical. It was shown through fractal analysis that the minimum dimension for the mod 8 problem was 0.1468 less than the original problem's minimum dimension and that the mod 8 problem's maximum dimension differed by 0.1504 from the original problem's. The null hypothesis established during this research project was accepted, and as future works, the researcher will describe a generalized method to find the indicator functions used for Collatz problems mod k.

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