The Smallest Lattice Cube Containing Nontrivial Band Knots

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In our work, we studied closed lattice bands of unit squares in the n×n×n lattice cubes. We wrote a computer program to find all such bands in the 2×2×2 lattice cube. We were able to identify each of such bands as an annulus or a Möbius strip. We consider symmetric ones as duplicates. Eliminating duplicates, we obtained 439 geometrically distinct closed bands. For each of these bands, we take the sequence of coordinates of the center of the squares to identify the knot type using the software ‘Knotplot’. They are all trivial knots. We proved that no nontrivial knots exist in the 3×3×3 lattice cube in the form of closed lattice bands. Furthermore, we found the trefoil knot in the 4×4×4 lattice cube and the figure eight knot in the 5×5×5 lattice cube.

Awards Won: