

Analogues of the Robin-Lagarias Criteria for the Riemann Hypothesis

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Robin's criterion states that the Riemann hypothesis is equivalent to $\sigma(n) < \exp(\gamma) n \log \log n$ for all integers $n > 5040$, where $\sigma(n)$ is the sum of divisors of n and γ is the Euler-Mascheroni constant. We prove that the Riemann hypothesis is equivalent to the statement $\sigma(n) < 0.5 \exp(\gamma) n \log \log n$ for all odd numbers $n > (3^4)(5^3)(7^2)(11)\dots(67)$.

Lagarias's criterion for the Riemann hypothesis states that the Riemann hypothesis is equivalent to $\sigma(n) < H(n) + \exp(H(n)) \log(H(n))$ for all integers $n > 1$, where $H(n)$ is the n th harmonic number. We establish an analogue to Lagarias's criterion for the Riemann hypothesis by creating a new harmonic series $h(n) = 2H(n) - H(2n)$ and demonstrating that the Riemann hypothesis is equivalent to $\sigma(n) < 3n/\log n + \exp(h(n)) \log(h(n))$ for all odd $n > 1$. We prove stronger analogues to Robin's inequality for odd squarefree numbers. Furthermore, we find a general formula that studies the effect of the prime factorization of n and its behavior in Robin's inequality.