Analogues of the Robin-Lagarias Criteria for the Riemann Hypothesis

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Robin’s criterion states that the Riemann hypothesis is equivalent to sigma(n) < exp(gamma) n log log n for all integers n > 5040, where sigma(n) is the sum of divisors of n and gamma is the Euler-Mascheroni constant. We prove that the Riemann hypothesis is equivalent to the statement sigma(n) < 0.5exp(gamma) n log log n for all odd numbers n > (3^4)(5^3)(7^2)(11)...(67). Lagarias’s criterion for the Riemann hypothesis states that the Riemann hypothesis is equivalent to sigma(n) < H(n) + exp(H(n))log(H(n)) for all integers n > 1, where H(n) is the nth harmonic number. We establish an analogue to Lagarias’s criterion for the Riemann hypothesis by creating a new harmonic series h(n) = 2H(n) - H(2n) and demonstrating that the Riemann hypothesis is equivalent to sigma(n) < 3n/log n + exp(h(n))log(h(n)) for all odd n > 1. We prove stronger analogues to Robin’s inequality for odd squarefree numbers. Furthermore, we find a general formula that studies the effect of the prime factorization of n and its behavior in Robin’s inequality.