On a Variation of the Witsenhausen Problem Concerning Maximal pi/2-Avoiding Spherical Sets

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The problem of finding the largest pi/2-avoiding spherical set first appeared in the AMS monthly journal in 1974. H. S. Witsenhausen asked readers to determine alpha(n)=(|U|)/(|S^(n-1) |), where U is the largest pi/2-avoiding set in the n dimensional sphere S^(n-1), that is, the largest set on the sphere in n dimensions containing no orthogonal vectors. We look at a variation of this problem: determine alpha(n,k)=(|U|)/(|S^(n-1) |), where U is the largest set on the n dimensional sphere S^(n-1) that contains no k mutually orthogonal vectors. We begin by specializing the problem to the case n=k=3 by considering spherical sets that avoid three mutually orthogonal vectors in 3D in an attempt to determine alpha(3,3) which allows for more interesting configurations and visualization. We find the largest spherical subset U not containing three mutually orthogonal vectors in the following cases: (i) U is a large double cap; (ii) U is a centered band; or (iii) U is a wedge shape. We then prove a generalized lower bound for α(n,k) for any n>/=3. By taking U, the union of two double caps and a centered band, we increase the area further. The measures of these sets give a provable lower bound for α(3,3). Finally, by smoothing the band using computational results, we find a configuration created by the intersection of eliptical cylinders with the sphere that gives an even better lower bound for alpha(3,3). We conjecture this to be the largest spherical set for alpha(3,3), and one that can be generalized to higher dimensions.