Diophantus Equations and Partially Ordered Sets

Gupta, Addea (School: Sanskriti School)

In Alzer, Horst and Luca's paper it is shown that the Diophantine equation (k!)^n+k^n= (n!)^k+n^k only has the trivial solution n=k, and (k!)^n- k^n= (n!)^k- n^k only has the solutions n=k, (n, k) = (1,2), and (2,1). In this article we find all solutions of the Diophantine Equations a_1!a_2!⋯a_n!±a_1a_2⋯a_n=b_1!b_2!⋯b_k!±b_1b_2⋯b_k, where a_i majorizes b_i. Furthermore we find a sufficient condition on a function f: N?R+ to guarantee that f gives a monotone function on the POSET of all finite sequences of natural numbers. We then use that to solve other Diophantine equations involving factorials and generalize the results of Sàndor's paper. We also explore similar Diophantine Equations for the Fibonacci Sequence and other sequences of natural numbers given by linear recursions of the form A_(n+2)=aA_(n+1)+bA_n.

Awards Won:

Third Award of $1,000