The Contraharmonic Mean: Connections, Relations to Number Theory, Possible Generalizations

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We often meet the well-known mathematical means: the arithmetic, geometric and harmonic means. They appear in inequalities, extremum problems, geometric connections. However, the ancient pythagoreans knew multiple means and they developed a common method for creating them. The contraharmonic mean belongs to the means obtained by this method. In my project I study this mean: its connections, relations to number theory and possible generalizations. For example the contraharmonic mean is connected to the pythagorean triples. I proved the related theorem using a geometric method. This method is also eligible for proving geometrically the lack of isotony in the case of the contraharmonic mean and for presenting its connection to the other means. The Lehmer-means are obtained by generalizing the contraharmonic mean. There is a theorem concerning the isotony of the Lehmer-means that I proved using a graphical method. I collected some examples for the occurrence of the Lehmer-means, among which there is an application related to engineering. By further generalization of the Lehmer-means are obtained the Gini-means that can be ranged into several families. I was able to graphically visualize these families and I found structures that present connections among means and mean families. I also studied the so-called contraharmonic numbers. I have conceived several necessary and sufficient conditions related to these numbers and their properties in the form of proven theorems and conjectures. I drew a parallel between the contraharmonic and harmonic numbers. The primary purpose of my project is to popularize the rather forgotten contraharmonic mean which is related to many curiosities.

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Fourth Award of $500