Simple Closed Geodesic Lines on Catalan Solids

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Closed geodesic lines on polyhedra are defined as locally shortest polylines not passing through vertices. In 2002 it was proven that the majority of convex polyhedra are free of simple (non-self-intersecting) closed geodesic lines. However, there are examples of simple closed geodesic lines on some categories of polyhedra (including my examples on the Archimedean solids). I focus on Catalan solids which are dual polyhedra to Archimedean solids, and the aim of my project is to prove or to refute the existence of simple closed geodesic lines on every Catalan solid. I studied polyhedra's nets (plane figures from which the polyhedron can be glued) because the geodesic line is straight on them. Firstly, I attempted to find two edges that are identified by folding. Next, I put two points, which will be one point after folding, on these segments. And if the straight line, which connects these points, goes inside the net, then there is a geodesic line. Using this method, I explored examples of simple closed geodesic lines on seven of thirteen Catalan solids. My method is effective and universal for any convex polyhedron but there is a problem: method is based on exhaustive search of nets. If I continue this search or invent a more optimal method I will prove or refute the existence of simple closed geodesic lines on Catalan solids. Outside of mathematics, my results are interesting for biologists who study a locomotion of some organisms (e.g. Physarum polycephalum) or organisms' societies (e.g. ant communities).

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