Visual Appearance of Extended Objects in Special Relativity

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The Lorentz contraction is a spontaneous measurement of an object's length. The difference between measuring and seeing is highlighted to compute the visual appearance of extended objects traveling at relativistic speeds. Given the initial position and velocity of a point (and the time "t" when it was seen by an observer) the point's "apparent position"—where it first emitted a photon—is calculated. Then the "apparent shape" of an extended object is computed given that different points emit light at different times such that all photons arrive simultaneously at "t". The actual shape is defined by an embedded 2-surface in the moving frame and the Poincare transformation is applied to determine the apparent shape in the observer's frame. To extend the same to accelerating objects (a previously unexplored problem), the hyperbolic trajectories of each point are derived using the Born-rigidity conditions. It is obtained that, in some cases, the apparent speed of a point exceeds that of light when approaching, similar to superluminal motion seen in space. By converting objects into mesh regions, the apparent shape of any object (including polytopes) is calculated. A generalized equation for the Doppler effect is derived and optical shading techniques are used to color objects based on received frequencies. For acceleration, objects with asymmetric visual distortions are simulated, and it is mathematically proven that an accelerating object will be invisible during certain times due to the Rindler horizon. The project is concluded by explaining how the same methodology may be applied to analyzing the appearance of an astrophysical jet. An overview of extending the same to curved spacetime is provided in the context of simulating the visual appearance of a receding galaxy.

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