Lebesgue Measure Preserving Thompson’s Monoid

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Real world mathematical models are often built to incorporate dynamical features like Lebesgue measure preservation, but the algebraic, group theoretical properties of dynamical systems have mostly been ignored in the present literature. In this project, for the first time I define Lebesgue measure preserving Thompson's monoid G by combining the algebraic features of Thompson's group F and measure preserving maps. I discover and prove algebraic and dynamical properties of G summarized next. I show that any continuous measure preserving map can be approximated by a G map with any required and the approximating map can be locally-eventually-onto (LEO) and achieve any entropy value of at least 2. I show that any G map is Markov and topological mixing is equivalent to LEO. I show that for maps in a specific subset of G periodic points exist with period 3, an essential feature of chaotic maps, and characterize periods of periodic points of other maps. Although G is not finitely generated, I construct a monoid by sets of equivalence classes such that the monoid is finitely generated and any G map is an element of an equivalence class in the monoid. I derive sufficient conditions for a continuous map to be topologically conjugate to a G map. The theorems of this project improve several state-of-the-art results in the literature, demonstrate an interesting interplay between algebraic and dynamical settings, and can find great applications in fluid dynamics, chaos theory, and group theory.

Awards Won:
Third Award of $1,000
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