The Structure of the Positive Monoid of Integer-Valued Polynomials Evaluated at an Algebraic Number

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Motivated by the characterization of symmetric monoidal functors between Deligne categories, the additive monoid R_+(x) is defined as the set of all nonnegative integer linear combinations of binomial coefficients (x choose n) for non-negative integers n. My project was concerned with the inquiry into the structure of R_+(alpha) for complex numbers alpha. I proved that this object is a ring if and only if a is an algebraic number that is not a nonnegative integer. I also gave two explicit representations of R_+(alpha), for both algebraic integers and general algebraic numbers alpha. One is in terms of inequalities for the valuations with respect to certain prime ideals and the other is in terms of explicitly constructed generators alpha_p. In particular, all algebraic integers generated by alpha are also contained in this ring. I also obtained a particularly good simplification of the structure of R_+(alpha) that works for all primes not dividing the discriminant of its minimal polynomial. This implies that a complete computationally effective characterization is obtained for every alpha, for all but finitely many prime numbers. Moreover, it leads to a particularly simple description of R_+(alpha) for both quadratic algebraic numbers and roots of unity alpha. I also recovered a result from Harman and Kalinov about the transitivity of R_+(alpha), only using combinatorial techniques, whereas their proof uses category theory.

Awards Won:

Fourth Award of $500

American Mathematical Society: Third Award of $500