Quadratization of ODEs: Monomial vs. Non-Monomial

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Quadratization is a transform of a system of ODEs with polynomial right-hand side into a system of ODEs with at most quadratic right-hand side via the introduction of new variables. It has been recently used as a preprocessing step for new model order reduction methods, so it is important to keep the number of new variables small. Several algorithms have been designed to search for a quadratization with the new variables being monomials in the original variables. To understand the limitations and potential ways of improving such algorithms, we study the following question: can quadratizations with not necessarily new monomial variables produce a model of substantially smaller dimension than quadratization with only new monomial variables? To do this, we restrict our attention to scalar polynomial ODEs. In our first result, we provide a necessary and sufficient condition on when one new variable is enough to quadratize. Secondly, we show that all degree 6 scalar polynomial ODEs can be quadratized with exactly two new non-monomial variables, an improvement on three new monomial variables. Based on these results, we observe that a quadratization with not necessarily new monomial variables can be much smaller than a monomial quadratization even for scalar ODEs. The main results of the paper have been discovered using computational methods of applied nonlinear algebra (Gröbner bases), and we describe these computations.

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