On Ramanujan's Identity for Odd Zeta Values and Its Generalization

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We begin by presenting an elementary proof of Ramanujan’s identity for odd zeta values. While Ramanujan's identity has been proved quite a few times earlier in the literature; there has been a lasting interest in finding an elementary and self-contained proof. Our proof solely relies on a Mittag-Leffler type expansion for hyperbolic cotangent function, manipulation of certain double sums and Euler’s identity for even zeta values. Next, we propose a general formula linearizing the convolution of Dirichlet series as the sum of Dirichlet series with modified weights; its specialization produces new identities and recovers several identities derived earlier in the literature, such as the two-parameter generalization of Ramanujan's identity for odd zeta values by B. Maji and collaborators, or the convolution of squares of Bernoulli numbers by A. Dixit and collaborators. A Ramanujan-type formula for Hurwitz zeta function and Bessel zeta function is also obtained. On the way, we also derive several important special cases of our transformations. Moreover, we show that at least one of the quantities zeta(5) and a series involving modified Bessel function is irrational.

Awards Won:
Fourth Award of $500
American Mathematical Society: First Award of $2,000