Functional Analysis of Parameterized Torus Knots

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Torus knots are knots that lie on the surface of a torus. Torus knots have a wide variety of applications in quantum mechanics, molecular chemistry, and topology. In this paper, a comprehensive study of the geometric and topological properties of torus knots is presented with emphasis on functional analysis. The torus knots were first defined by standardized parametric equations. Using the Wolfram Mathematica software system, I created interactive plots of each knot and calculated their dynamic properties: the first and second derivatives, curvature, and torsion. I also developed a program within Mathematica to visualize the tangent, normal, and binormal unit vector (TNB) frame of a torus knot in conjunction with a plot of its curvature and torsion. Connections to fractals and Lissajous curves were found, and further confirmation of the Fáry-Milnor theorem and fundamental theorem of space curves was established. Overall, the results of this project could be used in understanding the intrinsic properties of torus knots and applying their characteristics to analysis of dynamic systems.

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