Factorizations in Evaluation Monoids of Laurent Semirings

Zhu, Sophie (School: Williamsville East High School)

The question of the existence and uniqueness of integers is one of the oldest and most fundamental questions in mathematics. Studied by mathematicians such as Euler, Fermat, and Dedekind since 350 BC, recent findings are dedicated to multiplicative factorizations in specific sets called rings (such as the integers Z or Gaussian integers Z[i]). This project generalizes this study by fully characterizing additive factorizations in a broader class of rings called monoids. For positive real numbers a, let N_0[a, 1/a] be the semiring of real numbers f(a) with all f(x) in N_0[x, 1/x], where N_0 is the set of nonnegative integers and N_0[x, 1/x] is the semiring of Laurent polynomials with coefficients in N_0. In this project, we study various factorization properties of the additive structure of N_0[a, 1/a]. We characterize when N_0[a, 1/a] is atomic, then when N_0[a, 1/a] satisfies the ascending chain condition on principal ideals in terms of certain well-studied factorization properties. Finally, we characterize when N_0[a, 1/a] satisfies the unique factorization property and show that, when this is not the case, N_0[a, 1/a] has infinite elasticity.

Awards Won:
Fourth Award of $500
American Mathematical Society: Second Award of $1,000