A Group-Theoretic Generalization of Pythagoras' Theorem

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Pythagoras’ theorem is a foundational theorem in geometry, with many far-reaching implications in other fields. There are many proofs of Pythagoras' theorem - from geometry, trigonometry or as a consequence of Euler’s formula, exp(it) = cos t + i sin t. We show that a proof of Pythagoras’ theorem using Euler’s formula naturally lends itself to generalizations to more abstract spaces. Considering homomorphisms into the multiplicative operation of a ring, we show that a generalization of Pythagoras’ theorem must hold for the unique decomposition of the homomorphism into the sum of functions of odd and even parity. This work led us to consider objects, in abstract spaces, with similar properties to a right-angled triangle, and generalizations of Pythagoras’ theorem for such objects. From this, we show that Pythagoras' theorem can be seen as a special case of a more general theorem. Finally, we show how a restricted version of this more general theorem can be used to find properties of solutions to certain sets of differential equations.

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