# Power Sum Identities From Zigzag Chords

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A zigzag chord is a broken line with vertices on the circle and connecting in a zigzag way, and any two adjacent segments form a fixed certain angle. The initial observation of this project is that if we classify n segments of a zigzag chord into two groups by its parity, then the sums of the length of both groups are equal if n is odd, while the square sum of the length of both groups are equal if n is even. In this project, we greatly generalized this observation and gave a profile of power sum identities with respect to the number n of segments, the power k, and the number m of groups obtained from segments modulo the number m. Firstly, we determined the powers such that the power sum identities hold when segments are into 2 groups. We then generalized these results. We classified n segments of a zigzag chord into m groups according to their order modulo m starting from the first vertex. We found that the existence of the power sum identities relies not only on the power k but also on the common divisor of n and m. Our main results are: (1) If m is a divisor of n, there are power sum identities among these m groups of segments, where the powers are certain even numbers. (2) If the greatest common divisor of n and m is a half of m, there are power sum identities among m groups of segments, where the powers are certain odd numbers. In this project, we found new geometric invariants in zigzag chords. In other words, we can actually construct sets of real numbers, of which the power sums equal simultaneously for any consecutive odd or even powers.

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