Elementary Proofs of the Properties of the Sierpinski Gasket Graph

Tonapi, Anushka (School: Sri Kumaran Children's Home - CBSE)

My research looks at a self-similar triangular fractal known as the Sierpinski Gasket through the perspective of graph theory, by replacing intersections of line segments in the fractal with nodes in order to obtain a planar graph. The purpose of this project is to find novel ways to prove the Eulerianity, Hamiltonicity, bipartitecy and connectedness of the Sierpinski Gasket Graph, which are all properties related to the cycle structure and connectivity. The Sierpinski graph refers to the nth iteration of the fractal containing a finite number of edges and vertices. I proved its Eulerianity with an argument involving three lemmas, two of which I proved using contradiction and derivation, and one of which I used an induction argument for. I proved Hamiltonicity with an induction argument involving the fractal graph's recursive construction and self-similarity. I proved bipartitecy using a result from Skiena (1990) and a demonstration of even and odd cycle structures. Finally, I proved connectedness and 2-connectivity with a result from Wolfram MathWorld (Weinstein, E.) and a constructive argument. Therefore, I arrived at the conclusion that the Sierpinski Gasket graph is Eulerian, Hamiltonian, non-bipartite, and connected. The study of the cyclic structure and connectivity of the Sierpinski graph has several applications including the development of a Sierpinski wireless communication fractal antenna (Cohen 2003) and circuit design (Wong, Springer 2013). Eulerian paths and Hamiltonian cycles are used in DNA fragment assembly and genomic sequences respectively, and applying these properties to a fractal with a self-similar structure has several practical applications.

Awards Won: