Proof of the Complete Presence of a Modulo 4 Bias for the Semiprimes

Gyulev, Nikola (School: Model High School of Mathematics "Akademik Kiril Popov")

Dummit, Granville and Kisilevsky have recently shown that the proportion of semiprimes (products of two primes) not exceeding a given x, whose factors are congruent to 3 modulo 4, is more than a quarter when x is sufficiently large. They have also conjectured that this holds from the very beginning, that is, for all x bigger than 8. We give a proof for x bigger than or equal to 10^21 via an explicit approach based on their work. Together with their data for the remaining x, this results in a full proof of the conjecture. Our method consists of techniques from Analytic Number Theory, such as Partial Summation and cancellations with sums over primes with different remainders. We also rely on classical estimates for prime counting functions, as well as on very recent explicit improvements by Bennet, Martin, O'Bryant and Rechnitzer, which have wide applications in essentially any setting involving estimations of sums over primes.

Awards Won:
American Mathematical Society: One-Year Membership to American Mathematical Society to each winner (7 winning projects, up to 3 team members per project)
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