Ending States of a Special Variant of the Chip-Firing Algorithm

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We investigate a special variant of chip-firing, in which we consider an infinite set of rooms on a number line, some of which are occupied by violinists. In a move, we take two violinists in adjacent rooms, and send one of them to the closest unoccupied room to the left and the other to the closest unoccupied room to the right. We classify the different possible final states from repeatedly performing this operation. We introduce numbers R(N,\ell,x) that count labeled recursive rooted trees with N vertices, \ell leaves, and the smallest rooted path ending in x. We describe the properties of these numbers and connect them to permutations. We conjecture that these numbers describe the probabilities ending with different final states when the moves are chosen uniformly.

Awards Won:
Fourth Award of $500
American Mathematical Society: Honorable Mention and One-Year Membership to AMS (for 5 projects with up to 3 team members per project)
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