'String' Theory: How Long Is a Piece of String?

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For any given curve, integration is the customary method for determining the area under the curve. However, finding the actual length of the curve is a far more complex task and one rarely encountered in high school calculus courses. If the equation of the curved segment is known, a tedious integral based on Pythagoras' Theorem is required to find the exact value. For a non-uniform curve or string, however, integration may not be the most accurate or proficient method. The purpose of this investigation is to statistically analyse three mathematical techniques, and two variations in terms of their accuracy, efficiency and practicality in the real world, to determine the optimal technique for approximating the length of a curved path. For this investigation, three strings of precise 10cm, 20cm and 30cm lengths were scanned at least 15 times each in various orientations and analysed computationally to compare the effectiveness of the three techniques. The three techniques evaluated were the exact-value integral, with polynomial regression used to fit the string to a function; rectification, consisting of piecing together up to 3000 straight-line segments; and the Steinhaus Longimeter, a geometric application using three overlapping grids at 30° to each other. The integral was found to be the most accurate with an average absolute percentage error of 0.5%, surpassing rectification (0.837%) and the Steinhaus Longimeter (1.241%). The Steinhaus Longimeter technique was found to be less laborious, yet rectification emerged as the overall optimal technique, being the most efficient and comparable in accuracy and practicality.

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