# The Sinkhorn Limit of Positive 3 × 3 Matrices

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A positive square matrix is an n × n array of positive real number entries. The Sinkhorn-Knopp alternate minimization algorithm transforms these matrices into their respective doubly stochastic Sinkhorn limits. This effect is analyzed using both numerical and symbolic data. A modified version of Nathanson's explicit formula for the Sinkhorn limit of general positive 2 × 2 matrices is introduced. Then a methodology inspired by Nathanson and Zeilberger that utilizes Buchberger's algorithm to compute Gröbner bases in order to find an explicit formula for the Sinkhorn limit of general positive 3 × 3 matrices is discussed. After implementing this methodology, a desired explicit formula is provided and analyzed. Focal parts of this formula are symbolically analyzed for applications in finding explicit formulae for the Sinkhorn limit of n × n matrices where n>3.

**Awards Won:**