Strict Inequalities for the n-Crossing Number

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Knot theory is the mathematical study of tangled loops of string. Over the last 50 years, a number of applications have been found. For instance: molecules created with the same atoms, but shaped into different knots, have different and often unique properties. In cybersecurity, certain post quantum cryptography algorithms rely on the difficulty of classifying knots. For applications and theory, it's important to distinguish different knots. Yet no general method is currently known. In 2013, Adams introduced n-crossing projections—knot projections where each crossing consists of n intersecting strands—as a new approach for differentiating knots. I established a new relationship between n-crossing projections that gives the first understanding of n-crossing projections for n>=9. I also discovered the first knots where the 9-crossing projection behaves differently than the 7-crossing projection. Specifically, I found knots K with c_7(K)/=c_9(K). The n-crossing number c_n(K) of a knot K is the minimum number of crossings in an n-crossing projection of K. As n is any integer greater than one, c_n(K) gives infinitely many ways to distinguish knots. However, c_n(K) is difficult to compute, so inequalities relating different n-crossing numbers are needed. Previously, only three strict inequalities had been shown, and none included a crossing number more complicated than c_5(K). I have found and proven the fourth such equation. Specifically, I proved that c_9(K) <=c_3(K)-2 for all knots K that are not the trivial, trefoil, or figure-eight knot. I showed this inequality cannot be improved and used it to obtain previously unknown values for c_9(K).

Awards Won:
Second Award of $2,000
American Mathematical Society: One-Year Membership to American Mathematical Society to each winner (7 winning projects, up to 3 team members per project)
American Mathematical Society: First Award of $2,000
Mu Alpha Theta, National High School and Two-Year College Mathematics Honor Society: First Award of $ 1,500
National Taiwan Science Education Center: Taiwan International Science Fair Special Award is a trip to participate in the Taiwan International Science Fair