Detecting Causality in 2+1 Dimension Spacetimes Using Symplectic Quandles

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Spacetime is a mathematical model of our universe based on the general relativity theory. Our observable world in this spacetime is represented by causally related points, defined as points that a single light ray can traverse. Scientists use 3D Knots (or links) to study causally related points. Alexander-Conway polynomial is a commonly used invariant to distinguish such links from links of causally unrelated points. However, Allen and Swenberg identified an infinite series of potential sky-links, the only known counterexample that this polynomial cannot distinguish from the connected sum of Hopf links, a link of causally unrelated points. This project demonstrates that a symplectic quandle can distinguish all links in Allen-Swenberg series from the connected sum of Hopf links, demonstrating its capability to detect causality in conjunction with Alexander-Conway polynomial. Due to its connectedness property, a symplectic quandle offers additional information in the set of homomorphism from quandle to the knot; however, computing this set has been challenging as it requires solving a system of non-linear equations with a large solution space. This project develops an algorithm that solves this system in polynomial time. It also employs enhanced quandle counting polynomial that separately counts the cardinality of the image of each homomorphism to extract finer details. Further, it provides a combinatorial proof that if a symplectic quandle distinguishes the first Allen-Swenberg link then it can distinguish all links in the series. Our result will enable us to map our observable world in 2+1 dimension spacetime. This has the potential to extend to higher dimensions as the current research involving 3+1 dimension manifolds uses symplectic forms.

Awards Won:
American Mathematical Society: Honorable Mention and One-Year Membership to AMS (for 5 projects with up to 3 team members per project)