A Study on Arc Index of Theta Curves

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This study explored various general properties related to the theta curve's arc index. The arc index not only serves to classify spatial graphs but also plays a crucial role in computing a significant topological invariant called floer homology for knots. Therefore, the investigation of the theta curve's arc index is expected to greatly benefit future research related to theta curves. I first derived two lower bounds for the theta curve's arc index using the arc index of constituent knots, and also determined a lower bound in terms of crossing number by using a loop-link diagram, one of the representations of the arc presentation. Furthermore, by identifying the relationship between the Yamada polynomial and the number of caps and crossings in stacked tangles, I obtained a lower bound in terms of the highest and lowest degrees of the Yamada polynomial. This addresses the shortcomings of non-sharp bounds for cases such as Brunian theta curve. For the upper bound, I derived by proving the existence of an arc presentation with the number of half-planes equal to the crossing number plus 4. The spoking algorithm and a graph-theoretical lemma were used to justify its existence. Using the obtained bounds and the binding circle method, I determined the ranges of the arc index of pretzel theta curve and Kinoshita-Wolcott theta curve. Additionally, for all theta curves with crossing numbers of 7 or less, I computed the values of each bound, confirming that the newly derived bounds in the study are tight and thus meaningful.

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