# Counting Visible Points on Square Lattice by Arithmetic Functions With Asymptotic Behavior

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For a fixed positive integer b?N, let the b-sight lines be defined by f(x)=ax^b, for a?Q, with the origin O as the observing point (the position of the eyes). A point in the square lattice V(m)={ (i, j) | i,j?N, 1=i=m, 1=j=m} is said to be b-visible if it is the “first” point in V(m) that can be seen from the origin O through any sight line of the form f(x)=ax^b, for some a?Q. Let H_b (m) denote the total number of b-visible points in V(m). Our goal in this project is to enumerate H_b (m) and we show that it can be expressed by Möbius function. When b=1, due to symmetry, H_1 (m) can be further reduced to a very neat formula in terms of Euler function. Moreover, by a probability result in literature, we obtain a non-trivial asymptotic limit lim_{m?8}H_b (m)/^2 =1/?(b+1) where ?(s) is the Riemann-Zeta function. Finally, assuming that we now observe lattice points in V(m) from another square S(k)={(r,t) | 0=r=k,0=t=k}, not limited to just the origin O. To see every lattice in V(m), we show that it can be done from S(k) whose side length k is no more than A·v(p(m)), where p(m) is the number of primes less than or equal to m, and A=3/v(1- 8/9 ln(2.5) )˜6.965. Our result is novel and interesting as it links counting in combinatorics with arithmetic functions in number theory and asymptotic behavior from analysis.

**Awards Won:**