Symmetry, Fixed Points and Quantum Billiards: A Confluence of Ideas

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A mathematical billiard system is one composed of a planar or multidimensional surface and a moving object whose trajectory is defined by its initial position and speed vector, along with some reflection law. The study of these systems has yielded applications in quantum computing and physical modeling. The purpose of this research was to study a novel reflection law for billiard systems of regular n-gons in which an object starting on one of the sides and moving with any given slope reflects from a limited reflection towards the interior with a prescribed constant angle. I constructed the map for the position of such an object as a function of the starting point and the slope in the case of a regular triangle and square. In both cases, I have proved that the object's path converges to a stable inner rectangle or triangle, for all initial conditions and all slopes, on a square or triangular billiards respectively. I have also constructed a numerical model for the object's trajectory and determined equations yielding the speed of the object's convergence to a stable path. I have performed relevant simulations and discussed the resulting data in the context of the proposed model. The analysis of these systems can be used to develop independent control mechanisms for ground robots in delivery missions working within contested environments or simpler and smaller microchips in the form of quantum billiards with correlated electrons without the necessity of perfectly elastic collisions, offering an innovative way to encode information.

Awards Won:

First Award of $5,000

Dudley R. Herschbach SIYSS Award

American Mathematical Society: One-Year Membership to American Mathematical Society to each winner (7 winning projects, up to 3 team members per project)

American Mathematical Society: Second Award of $1,000

The University of Texas at Dallas: Scholarship awards of $5,000 per year, renewable for up to four years