Integration of Sequences

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In my research project I want to generalise the well-known concept of Riemann integration of functions on sequences in order to obtain a new tool for the investigation of sequences. To do this, I define an integral for sequences based on the Riemann integral and then investigate which sequences are integrable. I prove necessary and sufficient conditions for sequence integrability. It turns out that integrability for a sequence is a stronger condition than cesàro-convergence: Every integrable sequence is cesàro-convergent, but not every cesàro-convergent sequence is also integrable. As a sufficient criterion for integrability, we find a condition that ressembles to Lebesgue integration: If suitable preimages under the sequence are "measurable", meaning that we can assign them a kind of length, the sequence is integrable. For sequences with finitely many accumulation points, we prove that this criterion is also necessary.

Awards Won:
Mu Alpha Theta, National High School and Two-Year College Mathematics Honor Society: First Award of $ 1,500
Fondazione Bruno Kessler: Award to participate in summer school "Web Valley"
American Mathematical Society: One-Year Membership to American Mathematical Society to each winner (7 winning projects, up to 3 team members per project)
American Mathematical Society: Second Award of $1,000