Slice Rank Polynomial Method and Avoidance of Triangles in Fields

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We introduce the slice rank polynomial method with its origins from the Cap Set problem and focus on two applications. We look into colorings of R^n with no more than 2^O(n) monochromatic equilateral triangles of side length 1, with respect to the l_d metric for d=1,2,3. We derive an exponential lower bound on the number of required colours by simplifying an approach of Naslund for the analogous problem in which d=2 and such equilateral triangles are avoided at all. To do so, we extend a lemma by Croot-Lev-Pach to functions which are essentially polynomials, but can also contain absolute values of real numbers. We also consider a generalized version of the Cap Set problem, concerning avoiding isosceles triangles over finite fields. We extend recent ideas for right angles by Kovac and angles 45 and 60 degrees by Omar to isosceles triangles with any vertex angle. Some auxiliary ideas are the snake oil method for sums, results for existence of primes in intervals, bounds on binomial coefficients via the binary entropy function, and a greedy algorithm to represent a polynomial as a sum of not too many unit slice rank functions.