On the Construction and Applications of Infinite-Dimensional Lie Algebra Weight Systems in the Theory of Vassiliev Invariants

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We explore knots. If we allow self-intersections, we get Singular knots. To distinguish different knots, invariant functions are used. A function from the set of knots to a ring is called a Vassiliev Invariant if when extended to singular knots (via the Vassiliev-Skein Relation) it vanishes on all knots with a given number of self-intersections. They are of interest, because of a conjectured way to approximate any knot invariant using them. All Vassiliev Invariants turn out to satisfy a certain topological property (4T Relation), and because of that, we consider all invariants, that satisfy it - we call them framed weight systems. There is a powerful construction for invariants coming from finite-dimensional Lie Algebras, which turns out not to be strong enough to span all possible framed weight systems. The main goal of the paper is to determine whether an extension of the construction for infinite-dimensional Lie Algebras suffices. We examine in more detail the construction for Loop algebras (a family of infinite-dimensional Lie algebras), proving that it produces framed weight systems, which are multiplicative, and their images commute with all elements of the extension of the universal enveloping algebra of the corresponding Loop Algebra (the space where the construction lives). We also go into more detail about the weight system coming from the Loop algebra built upon the Lie algebra sl2, giving an argument, which is a strong indication that it differs from the weight system of sl2, as well as presenting several chord diagram identities, which provide convenient connections.

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