Linear Group of Seventh Chord Transformations

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Seventh chords are widely used in music. What are the relationships between different seventh chords? Can these relationships be explained mathematically? This research classifies transformations between seventh chords into six types using Euler's principle, namely U_1, V_1, W_1, U_2, V_2, and W_2. Each of these transformations is in matrix form. This research constructs a group generated by these transformations and names this group the General Transformation Group (GT). This research claims that every element in GT can be expressed in a unified form. This research also determines that GT is isomorphic to the semi-direct product of Z_2 and the internal direct product of three Z_12, and its center is isomorphic to the internal direct product of three cyclic groups of order 2. By using this group, we can examine the transformation between dominant sevenths and half-diminished sevenths from a generalized 4-chord perspective. This group also gives musicologists a theoretical guideline to seventh chords and their relationships and provides musicians and auto accompaniment with a clear picture of the usage of seventh chords. This research encourages further research into transformations of chords beyond seventh chords, aiming to broaden the applicability of the GT group's principles.

Awards Won:
Fourth Award of $500
American Mathematical Society: Honorable Mention and One-Year Membership to AMS (for 5 projects with up to 3 team members per project)