Fibonacci Analogues of Legendre's Formula and Fine's Theorem

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Legendre's Formula and Fine's Theorem use number theory to analyze combinatorics functions. Legendre's formula finds the exponent of a prime (p) dividing the factorial of any integer (n); Fine's Theorem quickly finds the number of binomial coefficients on row n of Pascal's Triangle that are not divisible by a prime p. Investigating the Fibonorial, a generalized form of the factorial based on the Fibonacci sequence, this research derives Fibonacci-analogues of Legendre's Formula and Fine's Theorem for Fibonorial/Fibonomial coefficients, which previously only analyzed the factorial/binomial coefficients. The study builds on Lengyel's Theorem regarding the relationship between the exponent of prime p dividing an integer n and dividing the nth Fibonacci number. A novel Fibonorial Legendre's Formula for the exponent of p dividing the nth Fibonorial was thus developed and proved. Additionally, employing the computational software system Mathematica, programs were designed for brute-force generation of the values of the analogous Fine's theorem. Identifying patterns within the lists of values generated, a conjectured Fibonomial Fine's Theorem was proposed and subsequently proven using a correspondence argument and the developed analogous Fibonorial Legendre's Formula. The research successfully derived a formula for the exponent of p dividing the nth Fibonorial and a validated formula for Fibonomial coefficients, expanding upon existing mathematical frameworks. This study contributes to understanding Fibonorial structures and their mathematical properties. Future work may extend to other factorial generalizations, related combinatorial problems, and explore broader applications in the increased computational efficiency of my formulae, tilings, and findings in nature.

Awards Won:
American Mathematical Society: Honorable Mention and One-Year Membership to AMS (for 5 projects with up to 3 team members per project)
National Security Agency Research Directorate : Third Place Award “Mathematics”